

Rezolvare Subiectul I

1. $3 \cdot 10 - 60 : 3 = 30 - 20 = 10.$

R: 10

2. $x = 100 \text{ lei}$

$\frac{25}{100} \cdot 100 = 25 \text{ lei} \Rightarrow 100 - 25 = 75 \text{ lei}$

R: 75

3. 986

R: 986

4. $A_{\text{cerc}} = 100\pi \text{ cm}^2.$

$A_{\text{cerc}} = \pi R^2$

$\Rightarrow \pi R^2 = 100\pi / : \pi$

$\Rightarrow R^2 = 100 \Rightarrow R = 10.$

R: 10

5.

$\angle (BC; A'C') \mid \Rightarrow \angle (B'C'; A'C') = \angle B'C'A' \Rightarrow$
 $BC \parallel B'C' \mid \Rightarrow m(\angle B'C'A') = 45^\circ.$

R: 45

6. $25^\circ - 18^\circ = 7^\circ.$

R: 7

Rezolvare SUBIECTUL al II-lea

2.

$$\overline{ab} = ?$$

$$\overline{ba} + 5(\overline{a} + 2\overline{b}) = 124 \quad \Rightarrow \quad \underline{10b} + a + 5a + \underline{10b} = 124$$

$$\overline{ba} = 10 \cdot b + a$$

$$20b + 6a = 124 \quad / : 2$$

$$10b + 3a = 62.$$

a, b - cifre

$$\Rightarrow \text{u.c.}(3a) = 2 \Rightarrow \boxed{a = 4} \quad \text{u.c.} = \text{ultima cifra}$$

$$3 \cdot 4 = 12$$

$$10b + 3a = 62$$

$$10b + 12 = 62 \Rightarrow 10b = 50 \Rightarrow b = \frac{50}{10} \Rightarrow \boxed{b = 5} \Rightarrow \boxed{\overline{ab} = 45}.$$

3.

$$\{x, y, z\} \text{ d.p. } \{2, 8, 10\} \Rightarrow \frac{x}{2} = \frac{y}{8} = \frac{z}{10} = k \Rightarrow \begin{cases} x = 2k \\ y = 8k \\ z = 10k \end{cases}$$

$$\boxed{mg = \sqrt{x \cdot y} = 12} \Rightarrow x \cdot y = 144 \Rightarrow 2k \cdot 8k = 144$$

$$\Rightarrow 16k^2 = 144 \Rightarrow k^2 = 144 : 16 \Rightarrow k^2 = 9 \Rightarrow$$

$$\Rightarrow \boxed{k = 3} \Rightarrow \begin{cases} x = 2k = 2 \cdot 3 = 6 \\ y = 8k = 8 \cdot 3 = 24 \\ z = 10k = 10 \cdot 3 = 30. \end{cases}$$

$$m_a = \frac{x + y + z}{3} = \frac{6 + 24 + 30}{3} = \frac{60}{3} \Rightarrow \boxed{m_a = 20}.$$



Rezolvare SUBIECTUL al II-lea

$$4 \quad a) (\sqrt{5} + \sqrt{3})^2 = (\sqrt{5})^2 + 2 \cdot \sqrt{5} \cdot \sqrt{3} + (\sqrt{3})^2 = 5 + 2\sqrt{15} + 3 = 8 + 2\sqrt{15}.$$

$$(1 - \sqrt{2})^2 = 1^2 - 2 \cdot 1 \cdot \sqrt{2} + (\sqrt{2})^2 = 1 - 2\sqrt{2} + 2 = 3 - 2\sqrt{2}.$$

$$a = (\sqrt{5} + \sqrt{3})^2 - \left(\frac{3\sqrt{5}}{\sqrt{3}} + \frac{5\sqrt{3}}{\sqrt{5}} \right) - (1 - \sqrt{2})^2 - (\sqrt{2})^2$$

$$a = 8 + 2\sqrt{15} - \left(\frac{8\sqrt{15}}{3} + \frac{5\sqrt{15}}{5} \right) - (3 - 2\sqrt{2}) - 2.$$

$$a = 8 + 2\sqrt{15} - \sqrt{15} - \sqrt{15} - 3 + 2\sqrt{2} - 2 = 3 + 2\sqrt{2} \Rightarrow$$

$$\Rightarrow a = 3 + 2\sqrt{2}$$

$$b) x = \frac{1}{a} + \frac{1}{b} - \frac{1}{ab} = \frac{1}{3 + 2\sqrt{2}} + \frac{1}{2\sqrt{2} - 3} - \frac{1}{(2\sqrt{2} + 3)(2\sqrt{2} - 3)} \Rightarrow$$

$$b = 2\sqrt{2} - 3 \Rightarrow x = \frac{3 - 2\sqrt{2}}{3^2 - (2\sqrt{2})^2} + \frac{2\sqrt{2} + 3}{(2\sqrt{2})^2 - 3^2} - \frac{1}{(2\sqrt{2})^2 - 3^2} \Rightarrow$$

$$\Rightarrow x = \frac{3 - 2\sqrt{2}}{9 - 4 \cdot 2} + \frac{2\sqrt{2} + 3}{8 - 9} - \frac{1}{8 - 9} \Rightarrow$$

$$\Rightarrow x = \frac{3 - 2\sqrt{2}}{1} + \frac{2\sqrt{2} + 3}{-1} - \frac{1}{-1} = 3 - 2\sqrt{2} - (2\sqrt{2} + 3) - (-1)$$

$$\Rightarrow x = 3 - 2\sqrt{2} - 2\sqrt{2} - 3 + 1 \Rightarrow x = 1 - 4\sqrt{2}$$

$$1,4 < \sqrt{2} < 1,5 \quad / \cdot (-4) \Rightarrow -5,6 > -4\sqrt{2} > -6 \quad / +1 \Rightarrow -4,6 > 1 - 4\sqrt{2} > -5$$

$$-5 < 1 - 4\sqrt{2} < -4,6 \Rightarrow -5 < x < -4,6$$

$$\left(-5; -\frac{23}{5} \right) \quad -\frac{23}{5} = -4,6 \quad \Rightarrow$$

$$\Rightarrow x \in \left(-5; -\frac{23}{5} \right).$$

$$5. \quad E(x) = (x+3)^2 - (x-1)(x+1) + x(x-5) - 10 = x^2 + 2 \cdot x \cdot 3 + 3^2 - (x^2 - 1^2) + x^2 - 5x - 10$$

$$E(x) = x^2 + 6x + 9 - x^2 + 1 + x^2 - 5x - 10 = x^2 + x = x(x+1) \Rightarrow$$

$$\Rightarrow E(n) = n(n+1) - n. \text{ par } (\text{produsul a 2 nr. naturale consecutive})$$

1.

$$B = AB = 12 \text{ cm.}$$

$$b = CA = 4 \text{ cm.}$$

$$h = AD = 8 \text{ cm.}$$

$$AE = 4 \text{ cm.}$$

$$AF = 6 \text{ cm.}$$

$$a) A_{ABCD} = 64 \text{ cm}^2.$$

$$b) m(\angle BCD) = ?$$

$$c) CE \perp FO$$

$$a) A_{ABCD} = \frac{(B+b) \cdot h}{2}$$

$$A_{ABCD} = \frac{(12+4) \cdot 8}{2} = \frac{16 \cdot 8}{2} \Rightarrow$$

$$\Rightarrow A_{ABCD} = 64 \text{ cm}^2.$$

$$b) AE \parallel CD$$

$$AE = CD = 4 \text{ cm} \mid \Rightarrow AECD \text{ paralelogram} \mid \Rightarrow m(\angle A) = 90^\circ$$

$$\Rightarrow AECD \text{ dreptunghi} \Rightarrow m(\angle ECD) = 90^\circ.$$

$$EB = AB - AE = 12 - 4 = 8 \text{ cm.}$$

$$CE = AD = 8 \text{ cm}$$

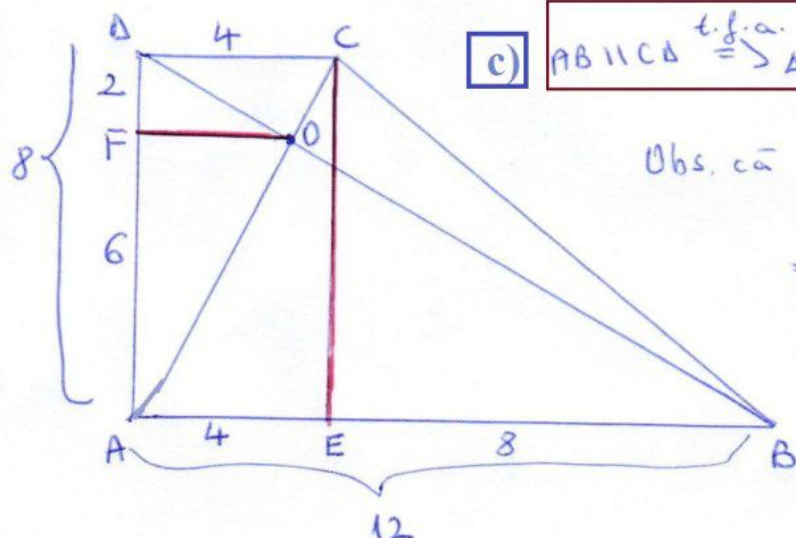
$$m(\angle CEB) = 90^\circ$$

$$\Rightarrow \triangle CEB \text{ dreptunghi} \Rightarrow \text{isoscel}$$

$$\Rightarrow m(\angle EBC) = m(\angle ECB) = 45^\circ.$$

$$\Rightarrow m(\angle BCD) = m(\angle ECB) + m(\angle ECD) = 45^\circ + 90^\circ \Rightarrow$$

$$\Rightarrow m(\angle BCD) = 135^\circ$$



c)

$$AB \parallel CD \xrightarrow{\text{t.f.a.}} \triangle AOB \sim \triangle COD \Rightarrow \frac{AO}{CO} = \frac{OB}{OD} = \frac{AB}{CD} = \frac{12}{4} = 3 \Rightarrow$$

$$\text{Obs. ca } \frac{AF}{DF} = \frac{6}{2} = 3$$

$$\Rightarrow \frac{AF}{DF} = \frac{AO}{CO} \xrightarrow{\text{Reciproc}} \text{Thales} \mid FO \parallel CD.$$

$$FO \parallel CD \mid \text{dar } CD \perp CE \mid \Rightarrow FO \perp CE.$$

REZOLVARE Subiectul III.2

2.

$$AB = 16 \text{ cm.}$$

$$BC = 8 \text{ cm.}$$

M - mijl. CD

$$VM \perp (ABCD)$$

$$a) P_{ABCD} = ?$$

$$b) d(V; AB) = 8\sqrt{2}$$

$$c) EF \parallel (VAB)$$

$$a) P_{ABCD} = 2L + 2l = 2 \cdot AB + 2 \cdot BC$$

$$P_{ABCD} = 2 \cdot 16 + 2 \cdot 8 = 32 + 16 = 48 \text{ cm.}$$

$$b) \text{Fie } MT \perp AB, T \in AB.$$

$$VM \perp (ABCD)$$

$$MT \perp AB$$

$$MT, AB \subset (ABCD)$$

$$\left. \begin{array}{l} VM \perp (ABCD) \\ MT \perp AB \end{array} \right| \xrightarrow{T3L} VT \perp AB \Rightarrow$$

$$\Rightarrow d(V; AB) = VT.$$

$$\left. \begin{array}{l} VM = 8 \text{ cm.} \\ MT = 8 \text{ cm.} \end{array} \right| \Rightarrow \Delta VMT \text{ dr. isoscel}$$

$$\Delta VMT: m(\angle M) = 90^\circ \xrightarrow{T.P.} VT^2 = VM^2 + MT^2 = 8^2 + 8^2$$

$$\Rightarrow VT^2 = 64 + 64 = 128 \Rightarrow VT = \sqrt{128} = \sqrt{64 \cdot 2} \Rightarrow VT = 8\sqrt{2} \text{ cm.}$$

$$c) M, O \text{ și } T \text{ sunt coliniare.}$$

E - mijlocul lui OM

$$ME = \frac{MO}{2} = \frac{4}{2} = 2 \text{ cm.}$$

$$\Rightarrow ET = 8 - 2 = 6 \text{ cm}$$

$$\Rightarrow \frac{ME}{ET} = \frac{2}{6} = \frac{1}{3}$$

$$\text{Dar } \frac{MF}{VF} = \frac{1}{3}$$

Proprietate
 $\Rightarrow EF \parallel VT$
Thales.

$$\left. \begin{array}{l} EF \parallel VT \\ VT \subset (VAB) \end{array} \right| \Rightarrow EF \parallel (VAB).$$

