

Subiectul I

1. $9 - 36 : \underbrace{(4+5)}_9 = 9 - \underbrace{36 : 9}_4 = 9 - 4 = 5.$

R: 5.

2. $\frac{x}{3} = \frac{4}{y} \Rightarrow xy = 4 \cdot 3 \Rightarrow xy = 12 \Rightarrow \frac{xy}{12} = \frac{12}{12} = 1.$

R: 1.

3. $-3 \cdot (-2) \cdot (-1) \cdot 0 \cdot 1 \cdot 2 = 0$

R: 0

4. $L_{arc} = 100\pi \text{ cm}$
 $L_{arc} = 2\pi R$ $\Rightarrow 2\pi R = 100\pi$
 $R = \frac{100\pi}{2\pi} = 50$

R: 50.

5.

$l = 6 \text{ cm}$

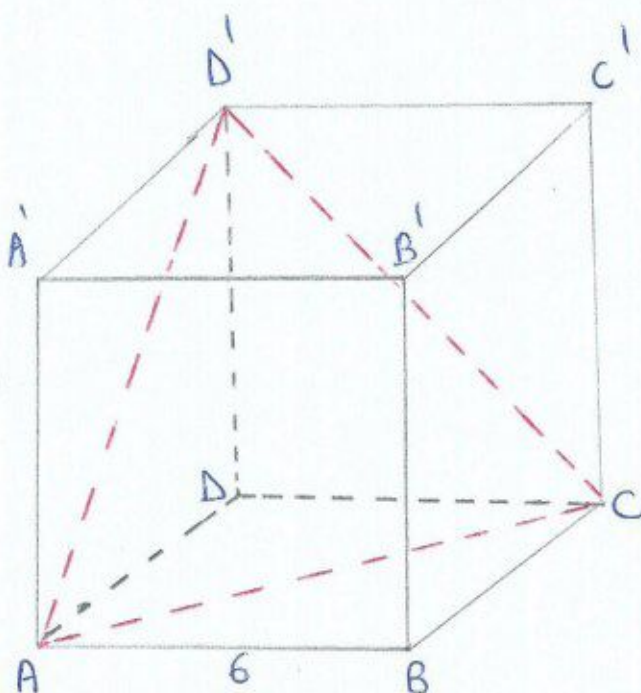
$P_{ACD'} = ?$

$AC = AD' = D'C = l\sqrt{2}$

$AC = AD' = D'C = 6\sqrt{2} \text{ cm}$

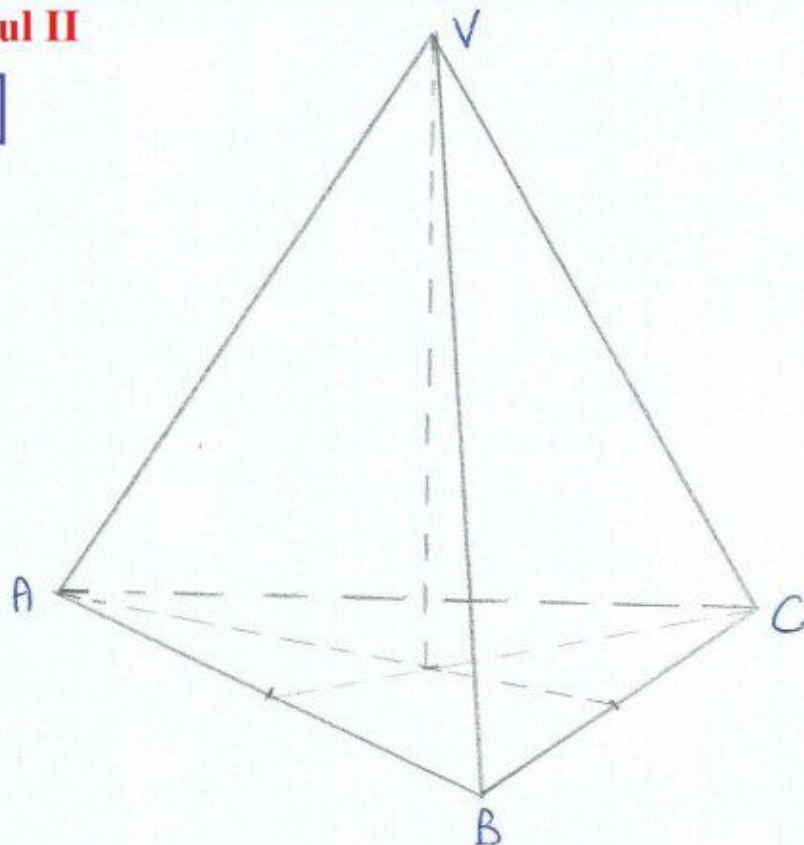
$P_{ACD'} = 3 \cdot 6\sqrt{2} = 18\sqrt{2} \text{ cm}$

R: $18\sqrt{2}$



6. $\frac{\partial a}{\partial x} 17 = 26^\circ \text{C}$
 $\frac{\partial a}{\partial x} 7 = 14^\circ \text{C}$
 $\frac{26 - 14}{17 - 7} = 12^\circ \text{C}$
R: 12

1.



2. $\frac{13}{x-7} \in \mathbb{N} \Rightarrow \underline{x-7 \in \Delta_{13}}; \Delta_{13} = \{1; 13\} \Rightarrow$

$\Rightarrow x-7 = 1 \Rightarrow x = 1+7 \Rightarrow \underline{x=8}.$

$\overset{\text{sau}}{x-7 = 13} \Rightarrow x = 13+7 \Rightarrow \underline{x=20}.$

3. $\underline{a+b=280}; a, b \in \mathbb{N}.$

$\underline{\frac{1}{3} \cdot a = \frac{1}{4} \cdot b} \Rightarrow \frac{a}{3} = \frac{b}{4} \Rightarrow a = \frac{3}{4}b$

$\begin{cases} a+b=280 \\ \uparrow \\ a = \frac{3b}{4} \end{cases} \Rightarrow \frac{3b}{4} + b = 280 \Rightarrow \frac{3b}{4} + \frac{4b}{4} = \frac{1120}{4} \quad / \cdot 4$

$\Rightarrow 3b + 4b = 1120 \Rightarrow 7b = 1120 \Rightarrow b = 1120 : 7$
 $\underline{b = 160}$

$b = 160 \Rightarrow a = 280 - 160 \Rightarrow \underline{a = 120}.$

Subiectul II

4.

$$a) \frac{\sqrt{2}}{\sqrt{2}-1} + \frac{2(\sqrt{2}-1)}{\sqrt{2}} = 4 \quad (=)$$

$$(\Rightarrow) \frac{\sqrt{2}(\sqrt{2}+1)}{(\sqrt{2})^2-1^2} + \frac{2\sqrt{2}(\sqrt{2}-1)}{(\sqrt{2})^2} = 4 \quad (=)$$

$$(\Rightarrow) \frac{\sqrt{2} \cdot \sqrt{2} + 1 \cdot \sqrt{2}}{2-1} + \frac{2\sqrt{2} \cdot \sqrt{2} - 2\sqrt{2} \cdot 1}{2} = 4 \quad (=) \quad \frac{2+\sqrt{2}}{1} + \frac{4-2\sqrt{2}}{2} = 4$$

$$(\Rightarrow) \frac{2(2+\sqrt{2}) + 4-2\sqrt{2}}{2} = 4 \quad (=) \quad \frac{4+2\sqrt{2} + 4-2\sqrt{2}}{2} = 4 \quad (=)$$

$$(\Rightarrow) \frac{8}{2} = 4 \quad (=) \quad \underline{4 = 4 \quad (A)}.$$

$$b) a = (\sqrt{5} + \sqrt{3})^2; \quad b = (\sqrt{5} - \sqrt{3})^2. \quad \begin{cases} (a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2 \\ (a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2 \end{cases}$$

$$a = (\sqrt{5})^2 + 2 \cdot \sqrt{5} \cdot \sqrt{3} + (\sqrt{3})^2 = 5 + 2\sqrt{15} + 3 = 8 + 2\sqrt{15}.$$

$$b = (\sqrt{5})^2 - 2 \cdot \sqrt{5} \cdot \sqrt{3} + (\sqrt{3})^2 = 5 - 2\sqrt{15} + 3 = 8 - 2\sqrt{15}.$$

$$mg = \sqrt{a \cdot b} \Rightarrow mg = \sqrt{(8+2\sqrt{15})(8-2\sqrt{15})} = \sqrt{8^2 - (2\sqrt{15})^2}$$

$$(a+b)(a-b) = a^2 - b^2$$

$$mg = \sqrt{64 - 4 \cdot 15} = \sqrt{64 - 60} = \sqrt{4} \Rightarrow \underline{mg = 2}.$$

5.

$$\underline{x+y=5}$$

$$E = x^2 + y^2 - 2xy - 3x - 3y + 2(2xy + 3)$$

$$E = x^2 + y^2 - 2xy - 3x - 3y + 2 \cdot (2xy) + 2 \cdot 3$$

$$E = x^2 + y^2 - 2xy - 3x - 3y + 4xy + 6$$

$$E = x^2 + y^2 + 2xy - 3(x+y) + 6$$

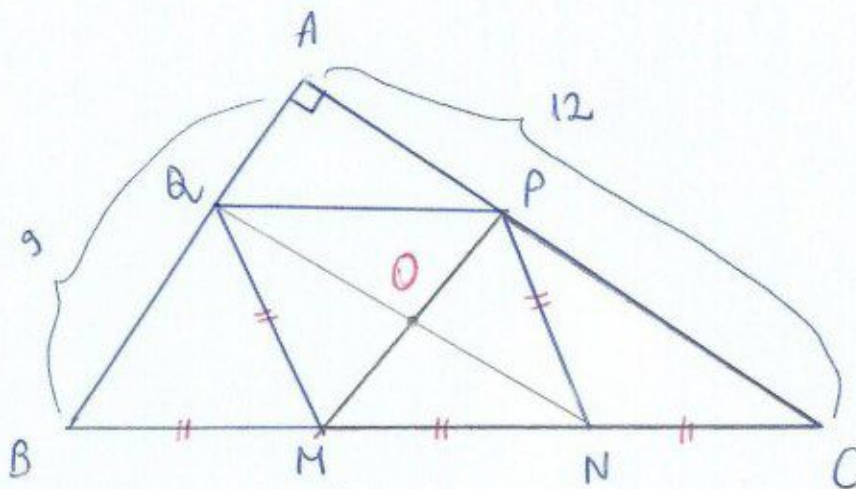
$$\text{Dar } x^2 + 2xy + y^2 = (x+y)^2$$

$$E = (\underline{x+y})^2 - 3(\underline{x+y}) + 6$$

$$E = 5^2 - 3 \cdot 5 + 6 = 25 - 15 + 6 = 10 + 6 \Rightarrow$$

$$\Rightarrow \underline{E = 16}.$$

1.



a) $\triangle ABC: m(\angle A) = 90^\circ \xRightarrow{T.P} BC^2 = AB^2 + AC^2$
 $BC^2 = 9^2 + 12^2 = 81 + 144 = 225$
 $BC = \sqrt{225} \Rightarrow BC = 15 \text{ cm.}$

$P_{ABC} = AB + BC + AC = 9 + 15 + 12 = 36 \text{ cm.}$

b) $BC = 15 \text{ cm}$
 $BC = BM + MN + NC \quad \left| \begin{array}{l} \Rightarrow BM = MN = NC = 15 : 3 = 5 \text{ cm.} \Rightarrow \\ \Rightarrow PN = QM = 5 \text{ cm.} \end{array} \right.$

$\triangle PMC: N - \text{mijlocul lui } BC \Rightarrow PN \text{ mediană}$
 Dar $PN = \frac{MC}{2} \left(5 = \frac{10}{2} \right) (A) \Rightarrow \triangle MPC \text{ dreptunghic}$
 $m(\angle MPC) = 90^\circ$

$\Rightarrow MP \perp AC$
 Dar $AB \perp AC \quad \left| \Rightarrow MP \parallel AB \Rightarrow \triangle CPM \sim \triangle CAB \Rightarrow \right.$
 $\Rightarrow \frac{CP}{CA} = \frac{PM}{AB} = \frac{CM}{CB} \Rightarrow \frac{CP}{12} = \frac{10}{15} \Rightarrow \frac{CP}{12} = \frac{10}{15} \Rightarrow$
 $\Rightarrow CP = \frac{12 \cdot 10}{15} = \frac{120}{15} = 8 \text{ cm.}$
 $\frac{PM}{AB} = \frac{10}{15} \Rightarrow \frac{PM}{9} = \frac{10}{15} \Rightarrow PM = \frac{9 \cdot 10}{15} = \frac{90}{15} = 6 \text{ cm.}$

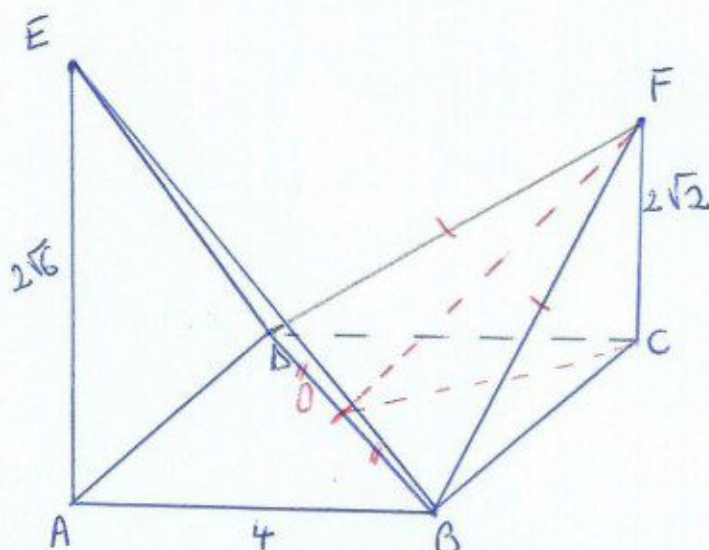
$A_{\triangle CPM} = \frac{c_1 \cdot c_2}{2} = \frac{PM \cdot CP}{2} = \frac{6 \cdot 8}{2} = \frac{48}{2} = 24 \text{ cm}^2.$

c) $QM \text{ mediană } \triangle QBN \text{ și } QM = \frac{BN}{2} \Rightarrow \triangle BQN \text{ dreptunghic în } Q.$

$\Rightarrow NA \perp AB$
 Dar $AB \perp AC \text{ și } MP \perp AC \quad \left| \Rightarrow MP \perp NQ. \right.$

$\triangle QMN \text{ isoscel } (QM = MN) \Rightarrow O - \text{mijlocul lui } QN$
 $\triangle PNM \text{ isoscel } (PN = MN) \Rightarrow O - \text{mijlocul lui } PM$
 $\Rightarrow \underline{MP \perp NQ} \quad \left| \Rightarrow \underline{MNPQ \text{ romb.}} \right.$

2.



a) ABCD pătrat cu $l = 4$ cm

$$AC = l\sqrt{2} = 4\sqrt{2} \text{ cm.}$$

b) $FC \perp (ABCD) \Rightarrow FC \perp BC$ și $FC \perp DC$ iar $\triangle FCB \equiv \triangle FCD \Rightarrow$
(FC-ec. comună și $[CB] \equiv [CD]$)

$\Rightarrow [FB] \equiv [FD] \Rightarrow \triangle FBD$ isoscel. Fie $FO \perp BD$, O - mijlocul lui BD.

$$\triangle FOC: m(\angle C) = 90^\circ \xrightarrow{T.P.} FO^2 = FC^2 + OC^2 = (2\sqrt{2})^2 + (2\sqrt{2})^2 = 4 \cdot 2 + 4 \cdot 2$$

$$OC = \frac{AC}{2} = 2\sqrt{2}$$

$$FO^2 = 8 + 8 = 16 \Rightarrow FO = \sqrt{16} = 4 \text{ cm.}$$

$$A_{FBD} = \frac{b \cdot h}{2} = \frac{BD \cdot FO}{2} = \frac{4\sqrt{2} \cdot 4}{2} = \frac{16\sqrt{2}}{2} = 8\sqrt{2} \text{ cm}^2.$$

c) $EA \perp (ABCD)$
 $AO \perp BD$
 $AO, BD \subset (ABCD)$ $\left| \begin{array}{l} T3 \perp \\ \Rightarrow EO \perp BD \end{array} \right.$

$$\left. \begin{array}{l} (EBD) \cap (FBD) = BD \\ EO \perp BD \\ FO \perp BD \end{array} \right\} \Rightarrow \angle((EBD), (FBD)) = \angle(EO, FO)$$

$\triangle FCO$ dreptunghic isoscel ($m(\angle FCO) = 90^\circ$ și $FC = OC = 2\sqrt{2}$) $\Rightarrow$

$$\Rightarrow m(\angle FOC) = 45^\circ.$$

$$\triangle EAO: m(\angle A) = 90^\circ \xrightarrow{T.P.} EO^2 = EA^2 + AO^2 = (2\sqrt{2})^2 + (2\sqrt{2})^2 = 4 \cdot 2 + 4 \cdot 2 \Rightarrow$$

$$\Rightarrow EO^2 = 24 + 8 = 32 \Rightarrow EO = \sqrt{32} = \sqrt{16 \cdot 2} = 4\sqrt{2} \text{ cm.}$$

$$\Rightarrow AO = \frac{EO}{2} \Rightarrow m(\angle AEO) = 30^\circ \Rightarrow m(\angle EOA) = 60^\circ \Rightarrow$$

$$\Rightarrow m(\angle EOF) = 180^\circ - 60^\circ - 45^\circ = 180^\circ - 105^\circ = 75^\circ.$$