

Subiectul I

1. $25 - 25 : (2 + 3) = 25 - \frac{25}{5} = 25 - 5 = \underline{20}$.

2. $4^2 = \underline{16}$; $5^2 = \underline{25}$; $6^2 = \underline{36}$; $7^2 = \underline{49}$; $8^2 = \underline{64}$; $9^2 = \underline{81}$

=> 6 pătrate perfecte de două cifre.

3. $A \cap B = \{\emptyset\}$.

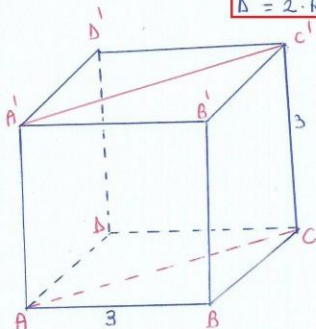
4. $L_{\text{cerc}} = 20\pi \text{ cm.}$ $\Rightarrow 2\pi R = 20\pi$
 $R = \frac{20\pi}{2\pi}$

$L_{\text{cerc}} = 2\pi R$

$R = 10 \text{ cm}$

$\Delta = 2 \cdot R = 2 \cdot 10 \Rightarrow \underline{\Delta = 20 \text{ cm.}}$

5.



$AC = 3\sqrt{2}$

$AC = 3\sqrt{2} \text{ cm. (L)}$

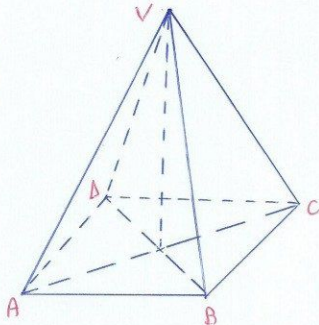
$CC' = 3 \text{ cm (l)}$

$A_{\text{acc}'A'} = L \cdot l$

$A_{\text{acc}'A'} = 3\sqrt{2} \cdot 3 = \underline{9\sqrt{2} \text{ cm.}}$

6. $6 + 7 + 5 + 4 = 13 + 9 = \underline{22 \text{ elevi.}}$

1.



2.

$$\overline{abc} = \overline{ab} + \overline{bc} + \overline{ca}, a \neq 0$$

$$\overline{abc} = c + 10b + 100a$$

$$\overline{ab} = b + 10a$$

$$\overline{bc} = c + 10b$$

$$\overline{ca} = a + 10c$$

$\rightarrow$

$$\Rightarrow c + 10b + 100a = \underline{b} + \underline{10a} + \underline{c} + \underline{10b} + \underline{a} + \underline{10c}$$

$$c + 10b + 100a = 11a + 11b + 11c$$

$$100a - 11a = \underline{11b} + \underline{11c} - \underline{10b} - \underline{c}$$

$$89a = b + 10c = \overline{cb} \Rightarrow \underline{a = 1}$$

$$\Rightarrow \underline{89 = \overline{cb}} \Rightarrow \underline{\underline{c = 8}} \\ \underline{\underline{b = 9}}$$

$$\Rightarrow \overline{abc} = 198$$

3.

$X = \text{lungimea brăcii}$

$$\underline{\text{I} \text{ zi}} = \frac{1}{2} \cdot X = \frac{X}{2}$$

$$\underline{\text{a} \text{ II} \text{ zi}} = \frac{1}{2} \cdot \text{I} \text{ zi} = \frac{1}{2} \cdot \frac{1}{2} \cdot X = \frac{X}{4}$$

$$\underline{\text{a} \text{ III} \text{ zi}} = 5 \text{ km}$$

$$\Rightarrow \begin{cases} X = \text{I} \text{ zi} + \text{II} \text{ zi} + \text{III} \text{ zi} \\ X = \frac{X}{2} + \frac{X}{4} + 5 \end{cases}$$

$$\frac{4X}{4} = \frac{2X}{4} + \frac{X}{4} + \frac{20}{4} \quad | \cdot 4$$

$$4X = 2X + X + 20$$

$$4X = 3X + 20$$

$$4X - 3X = 20 \Rightarrow \underline{\underline{X = 20 \text{ km.}}}$$

$$4. \quad a = \frac{1}{\sqrt{2}} + \frac{2}{\sqrt{8}} + \frac{3}{\sqrt{18}} + \frac{4}{\sqrt{32}}; \quad b = \frac{\sqrt{13^2 - 5^2}}{\sqrt{10^2 - 8^2}}$$

$$a) \quad a = \frac{1}{\sqrt{2}} + \frac{2}{2\sqrt{2}} + \frac{3}{3\sqrt{2}} + \frac{4}{4\sqrt{2}} \quad \left| \begin{array}{l} \sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2} \\ \sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2} \\ \sqrt{32} = \sqrt{16 \cdot 2} = 4\sqrt{2} \end{array} \right.$$

$$a = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$$

$$a = \frac{1+1+1+1}{\sqrt{2}} = \frac{4}{\sqrt{2}} = \frac{4\sqrt{2}}{2} \Rightarrow a = 2\sqrt{2}.$$

$$b) \quad b = \frac{\sqrt{13^2 - 5^2}}{\sqrt{10^2 - 8^2}} = \frac{\sqrt{169 - 25}}{\sqrt{100 - 64}} = \frac{\sqrt{144}}{\sqrt{36}} = \frac{12}{6} = \frac{2}{1} = 2.$$

$$b = 2.$$

$$a^2 - b^2 = (2\sqrt{2})^2 - 2^2 = 4 \cdot 2 - 4 = 8 - 4 = 4.$$

$$a^2 - b^2 = 4.$$

$$(a-b)(a+b) = a^2 - b^2$$

$$5. \quad E(x) = x^3 + \underbrace{(x+1)^2}_{(a+b)^2 = a^2 + 2ab + b^2} + 2(x-3)(x+3) + 17$$

$$E(x) = x^3 + x^2 + 2 \cdot x \cdot 1 + 1^2 + 2 \cdot (x^2 - 9) + 17$$

$$E(x) = x^3 + x^2 + 2x + 1 + 2 \cdot (x^2 - 9) + 17$$

$$E(x) = x^3 + x^2 + 2x + 1 + 2x^2 - 18 + 17$$

$$E(x) = x^3 + 3x^2 + 2x = x(x^2 + 3x + 2)$$

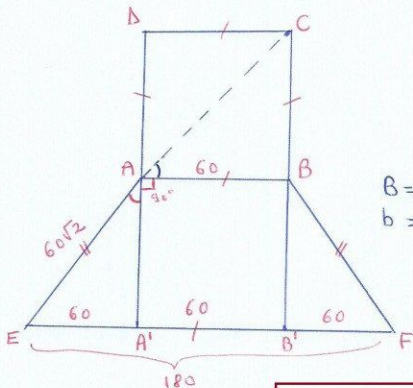
$$\begin{array}{c} \begin{array}{cc} 1 & 2 \\ \hline (x+1) & (x+2) \end{array} \end{array}$$

$$E(x) = x(x+1)(x+2)$$

$$E(m) = m \cdot (m+1) \cdot (m+2)$$

Produsul a 3 numere naturale consecutive este
multiplic de 6. ($E(m); 6$) $\forall m \in \mathbb{N}$.

1.



$$b = AB = 60 \text{ m}$$

a) Fic $AA' \perp EF$ e $BB' \perp EF$ $\Rightarrow EA' = B'F = \frac{\beta - b}{2} = \frac{180 - 60}{2} = \frac{120}{2}$
 $ABFE$ trapezoidal $\Rightarrow EA' = 60 \text{ m.}$

$$\begin{aligned} \Delta AA'E: m(\angle A') &= 90^\circ \xRightarrow{T.P} EA^2 = EA'^2 + AA'^2 \\ (60\sqrt{2})^2 &= 60^2 + AA'^2 \\ 3600 \cdot 2 &= 3600 + AA'^2 \\ AA'^2 &= 7200 - 3600 = 3600 \Rightarrow \\ \Rightarrow AA' &= \sqrt{3600} = \sqrt{60^2} \Rightarrow AA' = 60 \text{ m.} \end{aligned}$$

$$\Rightarrow d(A; EF) = AA' = 60 \text{ m.}$$

$$b) \underline{A_{\square} = l^2 = 60^2 = 3600 \text{ m}^2}; \underline{A_{\triangle} = \frac{(B+b) \cdot h}{2} = \frac{(180+60) \cdot 60}{2} = \frac{240 \cdot 60}{2} = 7200 \text{ m}^2}.$$

$$A_{\text{total}} = A_{\square} + A_{\triangle} = 3600 + 7200 = 10800 \text{ m}^2$$

c) $m(\angle EAC) = m(\angle EAA') + m(\angle A'AB) + m(\angle CAB)$

$$\Delta EAA' \text{ dr. isoscel} \Rightarrow m(\angle EAA') = 45^\circ$$

$ABB'A'$ pāthot $\Rightarrow m(\angle A'AB) = 90^\circ$.

$ABCD$ pătrat $\Rightarrow m(\angle CAB) = \frac{90^\circ}{2} = 45^\circ$

$$m(\angle EAC) = 45^\circ + 90^\circ + 45^\circ = 180^\circ \Rightarrow$$

$\Rightarrow E, A \text{ și } C$ sunt COLINIARE

2.

$$a) AB = 16 \text{ m} \Rightarrow \underline{BD = 2\sqrt{2} = 16\sqrt{2} \text{ m}} \Rightarrow \underline{BO = \frac{BD}{2} = 8\sqrt{2} \text{ m}}.$$

$$\angle(SB; (ABCD)) = \angle(SB; BD) = \angle SBD \quad \left| \begin{array}{l} O \in BD \end{array} \right| = \angle SBO.$$

$$\tan \widehat{SBO} = \frac{\text{cateta opusă}}{\text{cateta alăturată}} \Rightarrow \tan 60^\circ = \frac{SO}{\frac{BO}{\sqrt{3}}} \Rightarrow \sqrt{3} = \frac{SO}{8\sqrt{2}} \Rightarrow SO = 8\sqrt{6} \text{ m}.$$

$$b) \angle(VOM; (SOB)) = ?$$

$$\left. \begin{array}{l} (VOM) \cap (SOB) = \underline{VO} \\ \underline{VO} \perp \underline{OM} \\ OM \subset (VOM) \\ \underline{VO} \perp \underline{OB} \\ OB \subset (SOB) \end{array} \right\} \Rightarrow \angle(VOM; (SOB)) = \angle(OM; OB) = \angle MOB.$$

$$\underline{\Delta MOB \text{ dr. isoscel} \Rightarrow m(\angle MOB) = 45^\circ.}$$

c)

$$\underline{pr_{(SAB)} O = H} \Rightarrow \left. \begin{array}{l} \underline{OH \perp (SAB)} \Rightarrow OH \perp AB \\ \underline{SO \perp (ABCD)} \Rightarrow SO \perp AD \\ OH \cap SO = \{O\} \end{array} \right\} \Rightarrow \underline{AD \perp (OSH)}.$$

$$AD \perp (OSH) \Rightarrow AD \perp SH \Rightarrow \underline{SH \text{ m\^altime \^in } \Delta SAD}. (1)$$

$$\left. \begin{array}{l} OD \perp OA \\ OD \perp SO \\ OA \cap SO = \{O\} \end{array} \right\} \Rightarrow OD \perp (SOA) \Rightarrow OD \perp SA$$

$$\text{Dar } OH \perp (SAB) \Rightarrow OH \perp SA$$

$$OH \cap OD = \{O\}$$

$$\Rightarrow \underline{SA \perp (ODH)}$$

$$SA \perp (ODH) \Rightarrow SA \perp DH \Rightarrow \underline{DH \text{ m\^altime \^in } \Delta SAD} (2)$$

$$\text{Din (1) \^i (2)} \Rightarrow \underline{H \text{ este ortocentrul } \Delta SAD}.$$