

Subiectul I

$$1. \quad \frac{1}{2} \cdot \frac{2}{3} + \frac{8}{3} = \frac{1}{3} + \frac{8}{3} = \frac{9}{3} = \frac{3}{1} = \underline{3}.$$

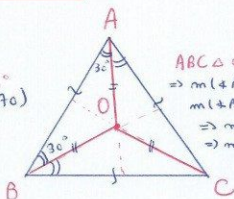
$$2. \quad \frac{10}{100} \cdot 20 = \frac{200}{100} = \frac{2}{1} = 2 \text{ lei}$$

(10% din pret)

$$20 \text{ lei} - 2 \text{ lei} = 18 \text{ lei.}$$

$$3. \quad \underline{n = 7.}$$

$$\begin{aligned} 4. \quad m(\angle AOB) &= \\ &= 180^\circ - m(\angle OBA) - m(\angle OAB) \\ &= 180^\circ - 60^\circ = \\ \Rightarrow \underline{m(\angle AOB) = 120^\circ.} \end{aligned}$$



$ABC \triangleq \text{echilateral} \Rightarrow$
 $\Rightarrow m(\angle A) = m(\angle B) = m(\angle C) = 60^\circ$
 $m(\angle ABC) = 60^\circ \Rightarrow$
 $\Rightarrow m(\angle ABO) = 30^\circ$
 $\Rightarrow m(\angle BAO) = 30^\circ$
 (din $m(\angle BAC) = 60^\circ$)

$$5. \quad \underline{AB + AD + AA' = 36 \text{ cm}}$$

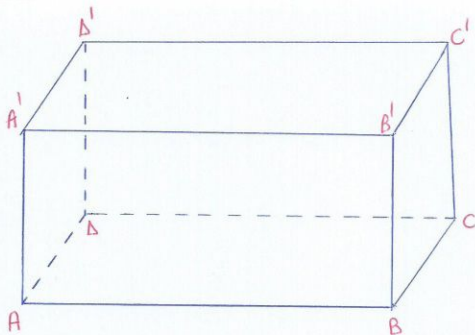
$$l + l + l = 36$$

$$3l = 36 \Rightarrow l = 36 : 3 = 12 \text{ cm.}$$

$$\Rightarrow \underline{AB = 12 \text{ cm.}}$$

$$6. \quad \underline{120 \text{ min} \rightarrow 80 \text{ km.}}$$

1.



2. $\overline{abc} : 5 \Rightarrow c \in \{0, 5\}$

$a + b + c = 22$

$c = 0 \Rightarrow a + b = 22$ cifra maximă 9 ($9 + 9 \neq 22$)
(fals)

$c = 5 \Rightarrow a + b + 5 = 22 \Rightarrow a + b = 22 - 5 = 17$

$a + b = 17$ ($9 + 8 = 17$ sau $8 + 9 = 17$) $\Rightarrow$

$\Rightarrow a = 9$
 $b = 8 \mid \Rightarrow \underline{\overline{abc} = 985}$

sau:
 $a = 8$
 $b = 9 \mid \Rightarrow \underline{\overline{abc} = 895}$

3. $x = \text{numărul de pagini ale cărții}$

I zi: $47\% \text{ din } x = \frac{47}{100} \cdot x \mid \Rightarrow \underline{x = \text{I zi} + \text{a II-a zi}}$

a II-a zi: 53 de pagini

$x = \frac{47}{100} \cdot x + 53 / \cdot 100$

$100x = 47x + 5300$

$100x - 47x = 5300$

$53x = 5300$

$x = \frac{5300}{53} \stackrel{153}{=} \frac{100}{1} = 100$

$x = 100 \text{ de pagini}$

Subiectul II

4.

$$x = \frac{1}{\sqrt{2}-1} + \frac{1}{\sqrt{2}+1}; \quad y = \sqrt{2} \cdot \left(\sqrt{2} + \frac{1}{\sqrt{2}} \right).$$

a)

$$x \cdot (\sqrt{8} - \sqrt{2}) = 4.$$

$$x = \frac{\sqrt{2}+1}{\sqrt{2}-1} + \frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{\sqrt{2}+1}{(\sqrt{2})^2-1^2} + \frac{\sqrt{2}-1}{(\sqrt{2})^2-1^2} = \frac{\sqrt{2}+1}{2-1} + \frac{\sqrt{2}-1}{2-1}$$

$$x = \frac{\sqrt{2}+1}{1} + \frac{\sqrt{2}-1}{1} = \underline{\underline{\sqrt{2}+1}} + \underline{\underline{\sqrt{2}-1}} = \underline{\underline{2\sqrt{2}}}$$

$$\begin{aligned} x \cdot (\sqrt{8} - \sqrt{2}) &= 2\sqrt{2} \cdot (\sqrt{8} - \sqrt{2}) = 2\sqrt{2} \cdot \sqrt{8} - 2\sqrt{2} \cdot \sqrt{2} \\ &= 2\sqrt{16} - 2\sqrt{4} = 2 \cdot 4 - 2 \cdot 2 = 8 - 4 = \underline{\underline{4}}. \end{aligned}$$

b)

$$y = \sqrt{2} \cdot \left(\sqrt{2} + \frac{1}{\sqrt{2}} \right) = \sqrt{2} \cdot \sqrt{2} + \sqrt{2} \cdot \frac{1}{\sqrt{2}} = 2 + \frac{\sqrt{2}}{\sqrt{2}} = 2 + 1 = \underline{\underline{3}}.$$

$$x = 2\sqrt{2} \Rightarrow x^2 - y = (2\sqrt{2})^2 - 3$$

$$y = 3 \Rightarrow x^2 - y = 8 - 3 = \underline{\underline{5}}.$$

5.

$$E(x) = (x^2 + x + 1)^2 - (x^2 + x)^2 - x^2$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$$

$$E(x) = (x^2)^2 + x^2 + 1^2 + 2 \cdot x^2 \cdot x + 2 \cdot x \cdot 1 + 2 \cdot x^2 \cdot 1 - (x^4 + 2 \cdot x^2 \cdot x + x^2) - x^2$$

$$E(x) = x^4 + x^2 + 1 + 2x^3 + 2x + 2x^2 - (x^4 + 2x^3 + x^2) - x^2$$

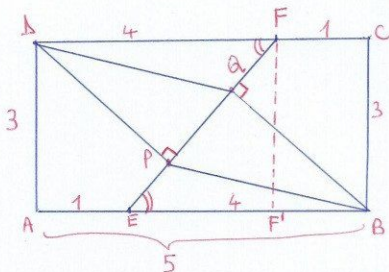
$$E(x) = \cancel{x^4} + \cancel{x^2} + 1 + \cancel{2x^3} + \underline{\underline{2x}} + \underline{\underline{2x^2}} - \cancel{x^4} - \cancel{2x^3} - \cancel{x^2} - \cancel{x^2}$$

$$\underline{\underline{E(x) = x^2 + 2x + 1 = (x+1)^2}}$$

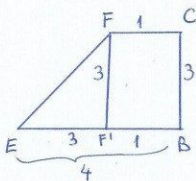
$$\underline{\underline{E(m) = (m+1)^2 \Rightarrow E(m) \text{ este pătrat perfect } \forall m \in \mathbb{N}.}}$$

Subiectul III

1.



a)



$$\text{Fie } FF' \perp EB \Rightarrow F'B = FC = 1 \text{ hm.} \\ FF' = CB = 3 \text{ hm.} \quad EF' = 4 - 1 = 3 \text{ hm.}$$

$$\Delta FFE: m(\angle F') = 90^\circ \xRightarrow{\text{T.P.}}$$

$$EF^2 = EF'^2 + FF'^2$$

$$EF^2 = 3^2 + 3^2 = 9 + 9 = 18$$

$$\Rightarrow EF = \sqrt{18} = \sqrt{9 \cdot 2} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2} \text{ hm.}$$

b)

ΔP drumul cel mai scurt $\Rightarrow \underline{AP \perp EF}$

$$\Delta EFF' \text{ dreptunghic isoscel } (EF' = F'F = 3 \text{ hm}) \Rightarrow m(\angle FEF') = m(\angle F'FE) = 45^\circ$$

$$\Rightarrow m(\angle PFA) = 45^\circ$$

(paralele, tăiate de o secantă)

$$m(\angle PFA) = 45^\circ \Rightarrow \Delta APF \text{ dreptunghic isoscel} \Rightarrow \underline{[AP] = [PF]}$$

$$\text{traseul } E-P-A = EP + PA = EP + PF = EF. \quad \underline{\text{Traseul } E-P-A \text{ și altele } EF \text{ sunt egale.}}$$

c)

$$\Delta P \perp EF \quad \underline{BQ \perp EF} \Rightarrow \underline{AP \parallel BQ} \quad (1)$$

$$\begin{array}{l} \Delta PFD \\ \Delta QEB \end{array} \quad \begin{array}{l} [AF] \equiv [EB] \\ \angle AFP \equiv \angle QEB \end{array} \quad \begin{array}{l} \text{i.u.} \\ \Rightarrow \end{array} \quad \begin{array}{l} \Delta PFD \equiv \Delta QEB \Rightarrow \\ \Rightarrow \underline{[AP] \equiv [BQ]} \quad (2) \end{array}$$

$$\underline{\text{Din (1) și (2) } \Rightarrow \Delta PBQ \text{ paralelogram.}}$$

2.

- a) E - mijlocul lui $[AB] \rightarrow [AE] \equiv [EB] \Rightarrow AE = \underline{EB} = \frac{AB}{2} = \frac{8}{2} = 4 \text{ cm.}$
 F - mijlocul lui $[BC] \Rightarrow \underline{BF} = FC = \frac{BC}{2} = \frac{8}{2} = 4 \text{ cm.}$

$$S_{\triangle BEF} = \frac{C_1 \cdot C_2}{2} = \frac{EB \cdot BF}{2} = \frac{4 \cdot 4}{2} = \frac{16}{2} = 8 \text{ cm}^2.$$

- b) Fie $\{O\} = BD \cap AC$.

$$\angle(VD, (ABC)) = \angle(VD, \Delta O) = \angle VDO.$$

$$\circ VD \perp (ABC)$$

$$\circ DO \in (ABC)$$

$$BD = 2\sqrt{2} = 8\sqrt{2} \text{ cm.} \Rightarrow DO = \frac{8\sqrt{2}}{2} = 4\sqrt{2} \text{ cm.}$$

$$\cos \widehat{VDO} = \frac{\text{cateta alăturată}}{\text{ipotenusa}} = \frac{DO}{VD} = \frac{4\sqrt{2}}{8} = \frac{\sqrt{2}}{2}$$

$$\Rightarrow m(\angle VDO) = 45^\circ.$$

- c) $\left. \begin{array}{l} \triangle MEB \\ \triangle MFB \end{array} \right\} \begin{array}{l} [EA] \equiv [FB] \\ \angle MBE \equiv \angle MBF \\ [MA] \equiv [MB] \text{ (latura} \\ \text{comună)} \end{array} \xrightarrow{\text{L.V.L.}} \underline{\triangle MEB \equiv \triangle MFB} \Rightarrow$

$$\Rightarrow m(\angle FMB) = \\ = m(\angle EMB) = 90^\circ$$

$$EM \perp VB \Rightarrow m(\angle EMB) = 90^\circ$$

$$m(\angle FMB) = 90^\circ \Rightarrow FM \perp VB$$

$$EM \perp VB$$

$$FM \perp VB$$

$$EM \cap FM = \{M\}$$

$$\Rightarrow \underline{VB \perp (EMF)}.$$