

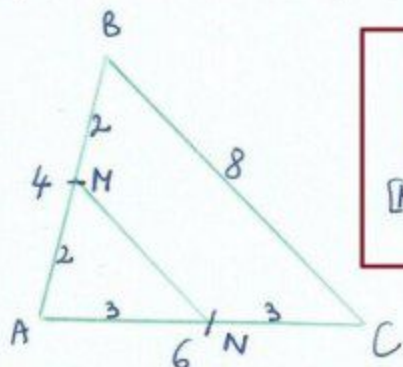
SUBIECTUL I

$$\begin{aligned} 1. \quad & \frac{(2^0 + 2^1 + 2^2) : (2^3 - 1)}{=} \\ & = \frac{(1 + 2 + 4) : (8 - 1)}{=} \\ & = \frac{7}{7} : \frac{7}{7} = \boxed{1} \end{aligned}$$

$$\begin{aligned} 2. \quad & \frac{a}{7} = \frac{5}{3} \\ & \frac{a+7}{7} = \frac{a}{7} + \frac{7}{7} = \frac{5}{3} + 1 = \frac{5}{3} + \frac{3}{3} = \boxed{\frac{8}{3}} \end{aligned}$$

$$3. \quad I = \{x \in \mathbb{R} / -5 \leq x \leq 3\} = \boxed{[-5; 3]}.$$

4.



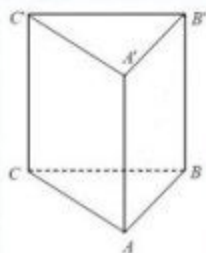
M - mijlocul lui AB $\Rightarrow AM = MB = \frac{4}{2} = 2 \text{ cm}$
 N - mijlocul lui AC $\Rightarrow AN = NC = \frac{6}{2} = 3 \text{ cm}$
 [MN] linie mijlocie în $\triangle ABC \Rightarrow MN = \frac{BC}{2} = \frac{8}{2} = 4 \text{ cm}$

$$P_{\triangle AMN} = AM + AN + MN = 2 + 3 + 4 = \boxed{9 \text{ cm}}.$$

$$\begin{aligned} 5. \quad & \angle(AD'; B'C) = \angle(BC'; B'C). \text{ Dar } BC' \perp B'C \text{ (diagonalele} \\ & \text{pătratului } BCC'B') \Rightarrow \\ & \angle(AD'; B'C) = \boxed{90^\circ}. \end{aligned}$$

$$6. \quad 26 + 25 + 27 = \boxed{78}.$$

1.



*de desenat o prismă triunghiulară regulată dreaptă ABCA'B'C'

2. $40 < m < 50$

$$\begin{cases} m : 6 = C_1 + 1 \\ m : 8 = C_2 + 1 \end{cases} \xRightarrow{\Delta = \hat{P} \cdot C + R} \begin{cases} m = 6 \cdot C_1 + 1 \quad / -1 \Rightarrow \\ m = 8 \cdot C_2 + 1 \quad / -1 \end{cases}$$

$$\Rightarrow \begin{cases} m-1 = 6 \cdot C_1 \\ m-1 = 8 \cdot C_2 \end{cases} \Rightarrow m-1 \in M[6; 8].$$

$$[6; 8] = ?$$

C.M.M.M.C

$$\begin{array}{r} 6 \overline{) 2} \\ 3 \overline{) 3} \\ 1 \end{array}$$

$$\begin{array}{r} 8 \overline{) 2} \\ 4 \overline{) 2} \\ 2 \overline{) 2} \\ 1 \end{array}$$

$$\begin{aligned} 6 &= 2 \cdot 3 \\ 8 &= 2^3 \end{aligned} \Rightarrow [6; 8] = 2^3 \cdot 3 = 8 \cdot 3 = \underline{24}.$$

$$\Rightarrow M_{24} = \{0, 24, 48, \dots\} \Rightarrow \underline{m-1 = 48} \Rightarrow m = 48 + 1 = \underline{49}.$$

$40 < m < 50$

3. $x = \text{suma inițială}$

$$\underline{x - \frac{2}{5} \cdot x - 13 = 8 / .5} \Leftrightarrow 5x - 2x - 65 = 40$$

$$3x - 65 = 40$$

$$3x = 40 + 65$$

$$3x = 105$$

$$x = \frac{105}{3} = 105 : 3 = 35$$

$$\underline{x = 35 \text{ lei.}}$$

$$4. \quad a = \sqrt{8}; \quad b = \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$a) \quad \frac{a+2}{a-2} = b$$

$$a = \sqrt{8} = 2\sqrt{2} \quad \begin{array}{r} 8 \\ 4 \overline{) 2} \\ 2 \overline{) 1} \end{array} > 2 \rightarrow 2\sqrt{2}$$

$$b = \frac{\sqrt{2}+1}{\sqrt{2}-1} = \frac{(\sqrt{2}+1)(\sqrt{2}+1)}{(\sqrt{2})^2 - 1^2} = \frac{\sqrt{2}^2 + 2 \cdot \sqrt{2} + 1}{2-1} = \frac{2+2\sqrt{2}+1}{1} = \underline{3+2\sqrt{2}}$$

$$\begin{cases} a = 2\sqrt{2} \\ b = 3+2\sqrt{2} \end{cases}$$

$$\frac{a+2}{a-2} = \frac{2\sqrt{2}+2}{2\sqrt{2}-2} = \frac{\frac{1}{2}(\sqrt{2}+1)}{\frac{1}{2}(\sqrt{2}-1)} = \frac{\sqrt{2}+1}{\sqrt{2}-1} = b.$$

$$b) \quad a < b \Leftrightarrow 2\sqrt{2} < 3+2\sqrt{2} \Leftrightarrow 2\sqrt{2} - 2\sqrt{2} < 3 \Leftrightarrow 0 < 3 \quad (A)$$

$$5. \quad E(x) = \frac{(1+x)(1-x)}{1} + \frac{(x+2)^2}{1} - \frac{2(x+2)}{1}$$

$$E(x) = \frac{1^2 - x^2}{1} + \frac{x^2 + 2 \cdot x \cdot 2 + 2^2}{1} - \frac{2 \cdot x - 2 \cdot 2}{1}$$

$$E(x) = 1 - \cancel{x^2} + \cancel{x^2} + \underline{4x} + \underline{4} - \underline{2x} - \underline{4}$$

$$E(x) = 1 + 2x.$$

$$\begin{cases} E(a) = -1. \\ E(a) = 1 + 2a \end{cases} \Rightarrow 1 + 2a = -1 \Leftrightarrow 2a = -1 - 1$$

$$2a = -2$$

$$a = \frac{-2}{2}$$

$$\boxed{a = -1.}$$

SUBIECTUL III

1.

a) $AB = 5 \cdot l = 5 \cdot 2 = 10 \text{ cm}$

$P_{ABCA} = 4 \cdot 10 = 40 \text{ cm.}$

b) $A_{\text{dallr}} = l^2 = 2^2 = 4 \text{ cm}^2$

Sunt 12 $\square$ albe $\rightarrow A_{\text{pătrate albe}} = 12 \cdot 4 = 48 \text{ cm}^2$ / $\rightarrow$

$A_{ABCA} = AB^2 = 10^2 = 100 \text{ cm}^2$

$\Rightarrow A_{\text{pătrate albe}} = 48\% \text{ din } A_{ABCA}.$

c) $\triangle ANM : m(\angle A) = 90^\circ \xrightarrow{\text{T.P.}} NM^2 = AN^2 + AM^2$

$AN = 2 \cdot l = 2 \cdot 2 = 4 \text{ cm}$

$AM = 3 \cdot l = 3 \cdot 2 = 6 \text{ cm}$

$NM^2 = 4^2 + 6^2$

$NM^2 = 16 + 36 = 52$

$NM = \sqrt{52} = 2\sqrt{13} \text{ cm.}$

$$\begin{array}{r} 52 \overline{) 2} \\ 26 \overline{) 2} \\ 13 \overline{) 13} \\ 1 \end{array}$$

$\triangle NAP : m(\angle A) = 90^\circ \xrightarrow{\text{T.P.}}$

$NA = 3 \cdot l = 3 \cdot 2 = 6 \text{ cm}$

$AP = 2 \cdot l = 2 \cdot 2 = 4 \text{ cm}$

$NP^2 = NA^2 + AP^2$

$NP^2 = 6^2 + 4^2$

$NP = \sqrt{52} = 2\sqrt{13} \text{ cm.}$

$\triangle PBQ : m(\angle B) = 90^\circ \xrightarrow{\text{T.P.}} PQ^2 = PB^2 + BQ^2$

$PB = 3 \cdot l = 3 \cdot 2 = 6 \text{ cm}$

$BQ = 2 \cdot l = 2 \cdot 2 = 4 \text{ cm.}$

$PQ^2 = 6^2 + 4^2$

$PQ^2 = 52 \Rightarrow PQ = \sqrt{52} = 2\sqrt{13} \text{ cm.}$

$\triangle MQC : m(\angle C) = 90^\circ \xrightarrow{\text{T.P.}} MQ^2 = MC^2 + CQ^2$

$MC = 2 \cdot l = 2 \cdot 2 = 4 \text{ cm}$

$CQ = 3 \cdot l = 3 \cdot 2 = 6 \text{ cm}$

$MQ^2 = 4^2 + 6^2$

$MQ^2 = 52 \Rightarrow MQ = 2\sqrt{13} \text{ cm.}$

$NM = NP = PQ = MQ = 2\sqrt{13} \text{ cm.} \Rightarrow$

$NMQP$ romb $\Rightarrow MP \perp NQ$
(diagonalele
rombului).

2.

$$\begin{aligned} \text{a)} \quad \Delta VOA: m(\angle O) &= 90^\circ \xrightarrow{\text{T.P.}} VA^2 = VO^2 + AO^2 \\ 6^2 &= (3\sqrt{2})^2 + AO^2 \\ 36 &= 9 \cdot 2 + AO^2 \\ 36 &= 18 + AO^2 \Rightarrow AO = \sqrt{36 - 18} \\ AO &= \sqrt{18} \\ AO &= \sqrt{9 \cdot 2} = 3\sqrt{2} \end{aligned}$$

$$AO = 3\sqrt{2} \Rightarrow AC = 2 \cdot 3\sqrt{2} = 6\sqrt{2} \text{ m.}$$

$$\begin{aligned} AC = d_{\square} = l\sqrt{2} &\Leftrightarrow 6\sqrt{2} = l\sqrt{2} \Leftrightarrow l = 6 \text{ m} \\ &\Rightarrow \underline{AB = 6 \text{ m.}} \end{aligned}$$

$$\begin{aligned} \text{b)} \quad &\left. \begin{array}{l} (VAC) \cap (VBA) = VO \\ AC \perp VO, AC \subset (VAC) \\ BD \perp VO, BD \subset (VBA) \end{array} \right\} \Rightarrow \begin{array}{l} \angle((VAC), (VBA)) = \angle(AC, BD) \\ m(\angle(AC, BD)) = 90^\circ. \end{array} \end{aligned}$$

$$\begin{aligned} \text{c)} \quad &\left. \begin{array}{l} M - \text{mijlocul lui } [BV] \\ N - \text{mijlocul lui } [CV] \end{array} \right\} \Rightarrow \begin{array}{l} [MN] \text{ este linie mijlocie in } \Delta VBC \Rightarrow \\ \Rightarrow \underline{MN \parallel BC.} \end{array} \end{aligned}$$

$$\begin{aligned} \left. \begin{array}{l} MN \parallel BC \\ BC \parallel AD \end{array} \right\} &\Rightarrow \underline{MN \parallel AD} \Rightarrow \underline{\text{punctele } A, D, N, M \text{ COPLANARE}} \Rightarrow \\ &\Rightarrow \underline{DM \text{ si } AN \text{ sunt coplanare.}} \end{aligned}$$